4D: Tesseract

Solution: First the trick is to recognize what's special about the three words given. They all involve traversing a route through the tesseract in which each turn is in a new direction. From a shade perspective, each shade is traversed just once. PUSHY for example uses criss-cross, then white, then gray, then black. BRASH uses white, then black, then criss-cross, then gray. From a probability point of view, from any vertex you have four choices, then three, then two, and then one, hence 4 x 3 x 2 x 1 = 24 different routes possible. Since there are 16 vertices, this makes 16 x 24 = 384 possible five letter sequences. There are (I believe) only four other words possible: BELCH, PARTY, PUDGY, SHONE which fit the four clues given. These may take some time for an individual to find, but hopefully three or four teammates can all be searching at the same time to make it go faster.

These four words actually utilize every letter on the tesseract just once, and four of the letters get used twice: P, H, E and Y. When these letters are highlighted or circled in the blanks, the solution **HYPE** is spelled out vertically.

B E L C H
P A R T Y
P U D G Y
S H O N E

Note that hype is hinted at with "overly excited" and "ballyhoo." Also note, a tesseract is often referred to as a hypercube, although technically a hypercube is any n-dimensional cube where n > 3.